Bounded Weights Trajectories for Neural Network State Estimation

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Abstract. The most important fact on differential neural networks (DNN) dynamics is related to the weights time evolution. This is a consequence for the complex non-linear structure describing the weights matrix differential equations associated with the adaptive capability for this kind of neural networks. However, there is no any analytical demonstration about the weights stability when the DNN approach is considered. In fact, this is the main inconvenient to design real applications for differential neural network observers (DNNO), especially to propose adaptive control functions for uncertain non-linear systems. This paper deals with the stability proof for the weights dynamics using an adaptive procedure to adjust the weights ODE. Three different examples (two of them were realized by numerical simulations and the last one was carried out using real bio-filtering process data) demonstrated the suggested approach performance.

1. Introduction

Artificial Intelligence (AI) is a vast and complicated subject area. Like many others, it is dogged by terminology and unexplained mathematics. At a base level, AI concerns some numbers that are changed by algorithms over time to achieve a certain goal. Many paper and books have been written on intelligent control using neural networks (NN) [1], [2]. NN's and their universal approximation property and learning capability [3] have proven to be a powerful tool to control complex dynamically non-linear systems with parameter uncertainties. Exploiting the fact of being universal approximates, it may straightforwardly substitute unknown system uncertainties by Neural Networks schemes, which is defined by a specific mathematical model (continuous, discrete, etc.) but contains a number of unknown parameters ("weights") to be adjusted. Depending on the suggested neural network model, its free parameters could appear as a linear or non-linear element in the NN description, and they may be modified using differential or difference equations [4], [5]. This kind of adjustment algorithm, transforms the original problem into a nonlinear robust adaptive feedback one. The differential neural network (DNN) approach, exploiting the properties of the applied NN, permits to avoid many problems related to global extreme search (as is usual when backpropagtion NN are applied), converting the learning process to an

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adequate feedback design, [5]. If the mathematical model of a considered process is incomplete or partially known, the DNN-approach provides an effective instrument to attack a wide spectrum of problems such as identification, state estimation, trajectories tracking, etc [3].

Lyapunov's stability theory has been used within the neural networks for control literature since fifty years [5], [6]. This is the main tool to guarantee close-loop system performance. Even though there is a general trend to enlarge the nonlinear systems for which the aforementioned works can be applied. On this field results on stability, convergence to arbitrarily small sets and robustness to modelling imperfections and external perturbations of the closed-loop system, have been established, in order to provide a larger class of non-linear systems that can be treat by this technique. The problem of learning in automatic control systems has been studied in the past, especially during 60s. At that time, learning was analyzed as the estimation or approximation of the unknown quantities in a function, in similar way to other terms as adaptation and self-organizing. Later on, learning theory has become a research discipline in the context of machine learning, and more recently, in computational or statistical learning using stochastic principles [7]. In neural modelling, learning is a path in the space of control parameters driving the system toward phase transition and bifurcations [8]. Though statistic learning theory could provide efficient learning algorithms for a wide variety of problems: in the robust analysis and synthesis of control systems, e.g., [9]. However, the same kind of learning process is complex to understand and difficult to be applied in practical state estimation and control systems, which are mostly dynamic and deterministic by nature.

The main contribution of this paper deals with the justification of the boundedness property for the weights trajectories involved in the suggested DNN state estimator description. By Lyapunov technique, the stable behaviour for the matrix differential learning law (which is also derived along this study) is analyzed. Besides, the adjustable parameters involved in the weights dynamics are suggested to be designed like sigmoid function or the, so-called, inverse one with a predefined rate convergence. Three simple examples demonstrated the workability of the approach provided in this paper.

2. Differential Neural Observer

The class of systems to be treated during this work is described by the vector nonlinear differential equation

$$\dot{x}_t = f(x_t, u_t, t) + \xi_{1,t}, \quad y_t = Cx_t + \xi_{2,t} \tag{1}$$

where $x_t \in \Re^n$ is the system state, $y_t \in \Re^p$ is the system output $(p \leq n)$, $u_t \in \Re^m$ is the bounded control action $(m \leq n)$ belonging to the following admissible set $U^{adm} := \{u : \|u\|^2_{\Lambda_u} = u^T \Lambda_u u \leq v_0 < \infty\}$ $\Lambda_u = \Lambda_u^? > 0$. The output matrix $C \in \Re^{p \times n}$ is assumed to be a-priory known. The nominal closed-loop dynamics is quadratically stable for a fixed control $u_t^* \in U^{adm}$, that is, there exists a Lyapunov function $\bar{V}_t$ such that :

$$\frac{\partial \bar{V}_t}{\partial x} f(x_t, u_t^*, t) \leq -\lambda_1 \|x_t\|^2 < 0$$
$$\left\|\frac{\partial \bar{V}_t}{\partial x}\right\| \leq \lambda_2 \|x_t\|^2 < \infty, \quad \lambda_1, \lambda_2 > 0 \tag{2}$$

The vectors $\xi_{1,t}$ and $\xi_{2,t}$ represent the state and output deterministic bounded (unmeasurable) disturbances, i.e., $\|\xi_{j,t}\|^2_{\Lambda_{\xi_j}} < \Upsilon_j$, $\Lambda_{\xi_j} > 0, j=1,2$, and do not violate the existence of the solution to ODE (1). The nominal output system (without external perturbations $\xi_{2,t} = 0$) is uniformly observable [10], that is, the following (observability) matrix is no singular for any $t \geq 0$:

$$\tilde{O} := \nabla_x \left[C^T, [L_f(Cx_t)]^T, [L_f^2(Cx_t)]^T, ..., [L_f^{n-1}(Cx_t)]^T\right]^T \tag{3}$$

Here $L_f(\cdot)$ is the Lie derivative operator [10]. To ensure the uniqueness and the existence of the non-linear dynamics, it is supposed the class of non-linear functions in (1) satisfies the Lipschitz condition (uniform on t) on two first arguments, that is,

$$\|f(x,u,t) - f(y,v,t)\| \leq L_1\|x - y\| + L_2\|u - v\|$$
$$\|f(0,0,t)\|^2 \leq C_1; \ x, y \in \Re^n; \ u, v \in \Re^m; \ 0 \leq L_1, L_2 < \infty \tag{4}$$

The last assumption automatically implies the following cone-property

$$\|f(x_t, u_t, t)\|^2 \leq C_1 + C_2\|x_t\|^2 \tag{5}$$

Which is valid for any x, u and t. Notice that (1) always could be represented as

$$\dot{x}_t = f_0(x_t, u_t, t \mid \Theta) + \tilde{f}_t + \xi_{1,t}$$
$$\tilde{f}_t := f(x_t, u_t, t) - f_0(x, u, t \mid \Theta) \tag{6}$$

where $f_0(x,u,t \mid \Theta)$ is treated as a possible *"nominal dynamics"* which can be selected according to a designer desires and $\tilde{f}_t$ is a vector called the *"no modelled dynamics"*. Here the parameters Θ are subjected to adjustment in order to obtain the complete matching between the nominal and the non-linear dynamics. In view of (6) and the corresponding boundedness property, the following upper bound for the no modelled dynamics $\widetilde{f}_t$ takes place:

$$\left\|\widetilde{f}_t\right\|^2_{\Lambda_f} \leq \widetilde{f}_0 + \widetilde{f}_1\left\|x_t\right\|^2_{\Lambda^1_{\widetilde{f}}},\ \Lambda_f, \Lambda^1_{\widetilde{f}} > 0 \tag{7}$$

According to DNN approach [5], we will define the nominal dynamics as

$$\begin{gathered} f_0(x,u,t \mid \Theta) = A^0 x + W_1^0\sigma(x) + W_2^0\varphi(x)u \\ \Theta := \left[W_1^0, W_2^0\right],\ A^0 \in \Re^{n\times n}, W_1^0, W_2^0, \in \Re^{n\times n}, \sigma \in \Re^{l\times 1}, \varphi \in \Re^{n\times m} \end{gathered} \tag{8}$$

The activation vector-functions $\sigma_j(\cdot)$ and $\varphi(\cdot)$ are usually constructed with sigmoid function components

$$\begin{gathered} \sigma_j(x) := a_j\left(1 + b_j \exp\left(-\sum_{j=1}^{n} c_j x_j\right)\right)^{-1} \\ \varphi_{kl}(x) := a_{kl}\left(1 + b_{kl} \exp\left(-\sum_{j=1}^{n} c_{kl} x_l\right)\right)^{-1} \\ j = \overline{1,n},\quad k = \overline{1,n},\quad l = \overline{1,m} \end{gathered} \tag{9}$$

It is easy to prove that each component in the activation functions satisfies the following sector conditions

$$\left\|\sigma(x) - \sigma(x')\right\|^2_{\Lambda_\sigma} \leq l_\sigma \left\|x - x'\right\|^2_{\Lambda_\sigma},\quad \left\|(\varphi(x) - \varphi(x'))u_t\right\|^2_{\Lambda_\varphi} \leq l_\varphi v_0 \left\|x - x'\right\|^2_{\Lambda_\varphi} \tag{10}$$

2.1. DNN structure containing relay correction term

Lets introduce the adaptive state estimator based on DNN [11] as follows:

$$\begin{gathered} \frac{d}{dt}\hat{x}_t = A^{(0)}\hat{x}_t + W_{1,t}\sigma(\hat{x}_t) + W_{2,t}\varphi(\hat{x})u_t + \\ K_1\left[y_t - C\hat{x}_t\right] + K_2\frac{y_t - C\hat{x}_t}{\left\|y_t - C\hat{x}_t\right\|},\quad \hat{y}_t = C\hat{x}_t \end{gathered} \tag{11}$$

So, when $\hat{y}_t = C\hat{x}_t$, ODE (11) should be attended as a differential inclusion [12]. Here the weights matrices ($W_{i,t}, i = 1,2$) supply the adaptive behaviour to this class of observers. The non-linear weight *updating* (learning) law is described by the following differential equations

$$\begin{aligned} \dot{W}_{i,t} &= -k_{i,t}\Xi_t + 2^{-1}k_{i,t}^{-1}\dot{k}_{i,t}\widetilde{W}_{i,t}, \quad i=1,2, \quad j=1,2 \\ \Xi_t &:= P_1 N_\delta \Omega_{i,t} \Psi_t^T, \qquad \Psi_1 = \sigma(\hat{x}_t), \;\; \Psi_2 = \varphi(\hat{x})u_t \\ \Omega_{i,t} &:= \left(\delta^2 \Lambda_j + C^T \Lambda_{\xi_2} C\right) N_\delta P_1 \widetilde{W}_{i,t} \Psi_t + C^T e_t, \end{aligned} \tag{12}$$

Here, the matrix P is the positive definite solution for the Riccati equation

$$\begin{gathered} P\widetilde{A}^{(0)\bullet} + \left(\widetilde{A}^{(0)\bullet}\right)^T P + PRP + Q = 0 \\ \overline{A} := A^{(0)\bullet} - K_1 C \\ Q := \Lambda_1^{-1} + \Lambda_2^{-1} + \Lambda_\sigma + v_0 \Lambda_\varphi + Q_0 \\ R := W_1^0 \Lambda_\sigma^{-1} \left[W_1^0\right]^T + W_2^0 \Lambda_\varphi^{-1} \left[W_2^0\right]^T + \Lambda_{\bar{f}} + \Lambda_{\xi_1} + K_1 \Lambda_{\xi_2} K_1^T + K_2 \Lambda K_2^T \end{gathered} \tag{13}$$

The problem analyzed here could be stated as follows: *Under the conditions A1-A4 for any admissible control strategy* u_t belonging to U^{adm} *, selecting matrices* $A^{(0)}$, K_1, K_2 *and designing the update law (12) (including the selection of* $W_i^{(0)}, i=1,2$*) in such a way that the upper bound for the averaged estimation error* β *defined as*

$$\beta := \limsup_{t\to\infty} \frac{1}{t+\varepsilon} \int_{s=0}^{t} \left\|\hat{x}_s - x_s\right\|_{Q_0}^2 ds, \quad \varepsilon, Q_0 > 0 \tag{14}$$

would be, as less as possible.

Theorem 1. *If there exist positive definite matrices* $\Lambda_{\bar{f}}, \Lambda_{\xi_1}, \Lambda_{\xi_2}, \Lambda_\sigma, \Lambda_D, \Lambda_u, \Lambda_1, Q_0$ *and positive constants* δ, k, v_1 *such that the matrix Riccati equation (13) has positive definite solution, then the DNN observer (11) with any matrix* K_1 *guarantying that the close-loop matrix* $\widetilde{A}^{(0)\bullet}$ *is stable, that is,* $\widetilde{A}^{(0)\bullet} := \left(A^{(0)\bullet} + K_1 C\right)$ is Hurwitz *and* $K_2 = \lambda P_1^{-1} C^T, \lambda > 0$ *supplied by the learning law (12), provides the following upper bound for the state estimation process:*

$$\begin{gathered} \overline{\lim_{T\to\infty}} \frac{1}{T} \int_{t=0}^{T} \left\|\Delta_t\right\|_P^2 dt \le \rho_Q / \alpha_Q \\ \rho_Q := \widetilde{f}_0 + v_0 + \Upsilon_1 + 3\Upsilon_2 + 8\lambda\sqrt{n\Upsilon_2}, \;\; \alpha_Q := \lambda_{\min}\left(P_1^{-1/2} Q_0 P_1^{-1/2}\right) > 0 \end{gathered} \tag{15}$$

The proof of this theorem could be done using the ideas given in [5].

2.2. Stable trajectories for the weights dynamics

Once the upper bound for the estimation process has been derived, it is possible (independently) to consider the stability analysis on the learning laws obtained during the observer development. The study was conducted suggesting a new Lyapunov-like function V_1 for the weight $\widetilde{W}_{1,t}$ (and the corresponding V_2 to $\widetilde{W}_{2,t}$) and applying the conventional procedure to determine the bounded behaviour for the weights dynamics. This result could be abstracted in the following:

Theorem 2. *The weights time trajectories are stable (in Lyapunov sense).*

Proof. Let considering the dynamics for the weight matrix $\widetilde{W}_{1,t}$ and the following (suggested) Lyapunov function V_1.

$$V_1 := \tfrac{1}{2} tr\left\{\widetilde{W}_{1,t}^T \widetilde{W}_{1,t}\right\} + \tfrac{c}{4}\left[k_{1,t} - k_{1,\min}\right]_+^2, \quad [z_t]_+ := \begin{cases} z & z \geq 0 \\ 0 & z < 0 \end{cases} \tag{16}$$

Which immediately implies (by direct differentiation)

$$\begin{aligned} \dot{V}_1 &:= tr\left\{\widetilde{W}_{1,t}^T\left(-k_{1,t}\Xi_1 + 2^{-1}k_{1,t}^{-1}\dot{k}_{1,t}\widetilde{W}_{1,t}\right)\right\} + 2^{-1}c\dot{k}_{1,t}\left[k_{1,t} - k_{1,\min}\right]_+ \\ &\leq k_{1,t}\left|tr\left\{\widetilde{W}_{1,t}^T\Xi_1\right\}\right| + 2^{-1}\dot{k}_{1,t}\left(\frac{1}{k_{1,t}} tr\left\{\widetilde{W}_{1,t}^T\widetilde{W}_{1,t}\right\} + c\left[k_{1,t} - k_{1,\min}\right]_+\right) \end{aligned} \tag{17}$$

If $\dot{V}_1$ is desired to be non positive, the free parameter $\dot{k}_{1,t}$ could be adjusted to obtain the asymptotic behaviour for the DNN weights dynamics in the following manner

$$\dot{k}_{1,t} \leq -\frac{2k_{1,t}^2\left|tr\left\{\widetilde{W}_{1,t}^T\Xi_1\right\}\right|}{tr\left\{\widetilde{W}_{1,t}^T\widetilde{W}_{1,t}\right\} + ck_{1,t}\left[k_{1,t} - k_{1,\min}\right]_+} \tag{18}$$

The same approach could be apply to $\widetilde{W}_{2,t}$ structure, resulting the corresponding upper value for

$$\dot{k}_{2,t} \leq -\frac{2k_{2,t}^2\left|tr\left\{\widetilde{W}_{2,t}^T\Xi_2\right\}\right|}{tr\left\{\widetilde{W}_{2,t}^T\widetilde{W}_{2,t}\right\} + ck_{2,t}\left[k_{2,t} - k_{2,\min}\right]_+} \tag{19}$$

These two conditions avoid the finite time escape for the weights dynamics. This tool could be used specially when there is any possible feedback control using the Weights Dynamics like adaptive parameter.

2.3. Learning Laws Examples

In this subsection, two simple examples for adaptive learning laws are introduced using the next definition

$$s\left(\widetilde{W}_{j,t},e_t\right) := 2k_{j,t}^2\left|tr\left\{\widetilde{W}_{j,t}^T\Xi_j\right\}\right|\left[tr\left\{\widetilde{W}_{j,t}^T\widetilde{W}_{j,t}\right\}+ck_{j,t}\left[k_{1,t}-k_{1,\min}\right]_+\right]^{-1} \quad (20)$$

a) Sigmoid learning law. The first example uses the sigmoid representation:

$$k_{j,t} := k_{0,j}\left(1+a\left(\widetilde{W}_{j,t}^T,e_t\right)\exp\left(b_j t\right)\right)^{-1}+k_{\min,j},\quad k_{\min,j}>0 \quad (21)$$

The corresponding time history $\dot{k}_{j,t}$ could be easily derived

$$\dot{k}_{j,t} := -k_{j,t}a\left(\widetilde{W}_{j,t}^T,e_t\right)b_j\exp\left(b_j t\right)\left(1+a\left(\widetilde{W}_{j,t}^T,e_t\right)\exp\left(b_j t\right)\right)^{-1} \quad (22)$$

which itself implies

$$a\left(\widetilde{W}_{j,t}^T,e_t\right)\exp\left(b_j t\right)\left(k_{j,t}b_j-s\left(\widetilde{W}_{j,t}^T,e_t\right)\right)>s\left(\widetilde{W}_{j,t}^T,e_t\right) \quad (23)$$

Last inequality is fulfilled if and only if the weight dependent parameter $a\left(\widetilde{W}_{j,t}^?,e_t\right)$ is selected in such a way

$$a\left(\widetilde{W}_{j,t}^T,e_t\right)>s\left(\widetilde{W}_{j,t}^T,e_t\right)\exp\left(-b_j t\right)\Psi_j^{-1},\quad \Psi_j := k_{j,t}b_j-s\left(\widetilde{W}_{j,t}^T,e_t\right) \quad (24)$$

b) Inverse learning law with predefined rate convergence. The second adaptive learning law is described by the inverse learning law with predefined rate convergence:

$$k_{1,t} := k_0\left(1+a\left(\widetilde{W}_{1,t}^T,e_t\right)t^k\right)^{-1}+k_{\min},\quad k_{\min}>0,\quad a\left(\widetilde{W}_{2,t}^T,e_t\right)>0 \quad (25)$$

It is easy to proof the time derivative for this function is described like

$$\dot{k}_{1,t} = k_0 ka\left(\widetilde{W}_{1,t}^T,e_t\right)t^{k-1}\left[1+a\left(\widetilde{W}_{1,t}^T,e_t\right)t^k\right]^{-2},\quad k>1,\quad k\in\aleph \quad (26)$$

The free parameter $a\left(\widetilde{W}_{1,t}^T,e_t\right)$ should fulfil the following restriction

$$a\left(\widetilde{W}_{1,t}^T,e_t\right)\geq\frac{k_0-2s\left(\widetilde{W}_{1,t}^T,e_t\right)t^k\pm\sqrt{k_0^2-4k_0 s\left(\widetilde{W}_{1,t}^T,e_t\right)t^k}}{2s\left(\widetilde{W}_{1,t}^T,e_t\right)t^k}>0 \quad (27)$$

It is important to notice that the learning laws must be adjusted on-line, i.e. each change on the weights values implies the corresponding change in the parameter $a(\cdot)$, so the adaptive process is carried out in two steps: the first one with the learning law application and second, the modification in the learning law rates $k_{j,t}$.

3. Numerical Examples

Three different numerical examples are analyzed using the suggested method: a benchmark problem represented by a linear system, a chemical model dealing with the ozonation process to treat water and finally the data corresponding to a real biological process called biofilter method.

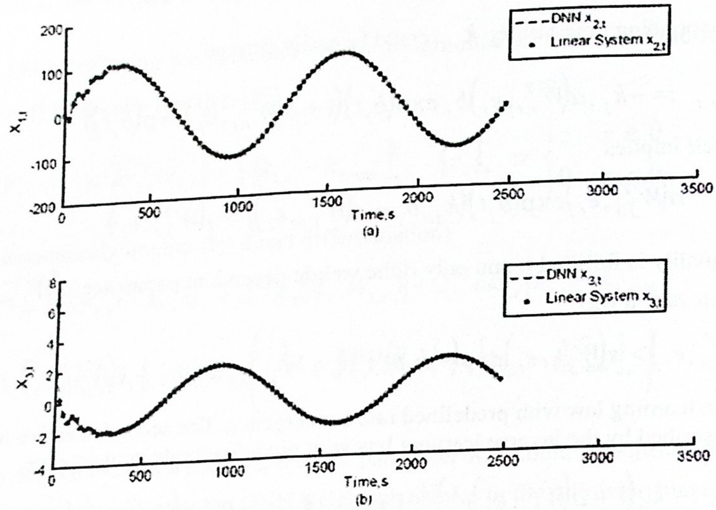

Figure 1. State estimation procedure for the linear system. a) State one non-parametric observation procedure and b) State three non parametric estimation procedures.

a) Benchmark problem: Linear System

Let consider the linear system given by:

$$A := \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -5 & -2 \end{bmatrix}, \; B := \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{28}$$
$$C^T := [1 \;\; 0 \;\; 0], \; u(t) := 3\sin(t) + WN(0.1)$$

Here $WN(0.1)$ is the standard pseudo-white noise signal generated by Matlab-simulink. This example is presented like a benchmark problem in order to prove the workability of this "adaptive" identification approach for simple systems. The estimation process was carried out considering the adaptive learning law ensuring the bounded property for the weights involved in the DNN structure. The DNNO application on the linear system () demonstrated the complete convergence between the state given by the linear system (x_t is assumed to be completely accessed by any physical mean) and the state given by the suggested approach (Fig. 1).

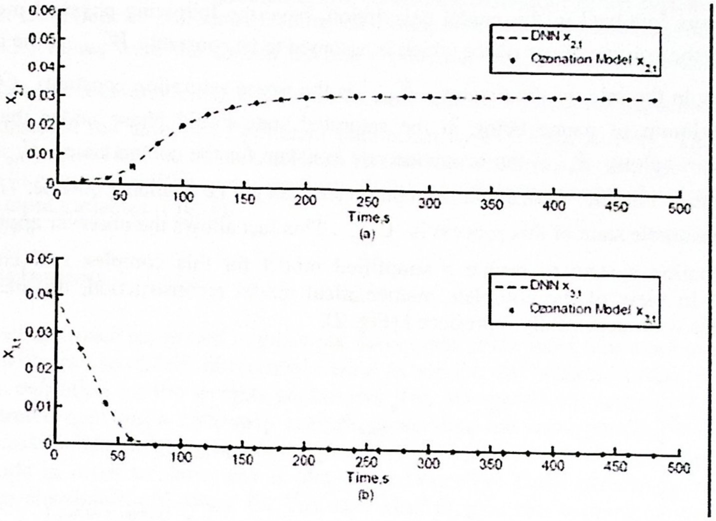

Figure 2. State estimation procedure for the ozonation system. a) Reconstructed state Q_t (dissolved oxygen) given by DNN observer and b) decomposition dynamics for the contaminant $C_1^{\prime}$ derived by non-parametric estimation procedure.

b) Ozonation reaction: mathematical model approach

Ozone is capable to oxidize a variety of organic materials in aqueous solution. The oxidation process by ozone involves the phenomenon of mass transfer with simultaneous chemical reaction. Based on the results obtained in [11], where the simple ozonation with i-component model mixture at the pH 7 has been treated. This process can be described using the following system of ODE:

$$\dot{C}_{gas}^{\prime} = \frac{1}{V_{gas}}\left[W_{gas}\left(C_{gas}^{in} - C_{gas}^{\prime}\right) - K_{sat}\left(Q_{\max} - Q^{\prime}\right) + k_1 C_1^{\prime} Q^{\prime}\right]$$

$$\dot{Q}^{\prime} = K_{sat}\left(Q_{\max} - Q^{\prime}\right) - k_1 C_1^{\prime} Q^{\prime}, \quad \dot{C}_1^{\prime} = -\frac{k_1 C_1^{\prime}}{V_{liq}} Q^{\prime} \tag{29}$$

where C'_{gas} is the ozone concentration in the output of the reactor (this is ozone which doesn't react with organic compounds dissolved in the solvent), Q_t is ozone dissolved in a liquid phase, C'_1 is the organic compound concentration at time t. The parameters involved in the model description, have the following physical meaning: V_{gas} is the volume of gas phase which is assumed to be constant, W_{gas} is the oxygen gas flow in the inlet of the reactor, K_{sat} is the ozone saturation constant, Q_{max} is the maximum of ozone being in the saturated state liquid phase under the given conditions below, k_1 is the ozonation rate constant for the contaminate, V_{liq} is the liquid phase volume. Notice that this component can not be available on-line. The only one measurable state of this process is C'_{gas}. This fact allows the observer application on ozonation system to derive a simplified model for this complex real chemical system. In view of the complete mathematical model reconstruction, the ozonation dynamics was successfully reproduced (Fig. 2).

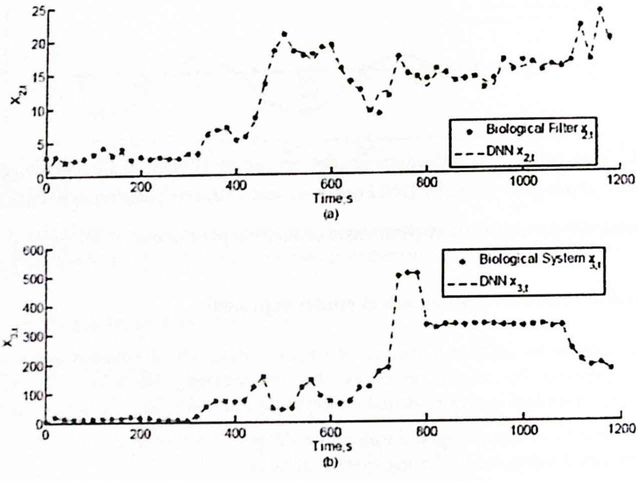

Figure 3. Observation procedure for bio-filtering method. a) Reconstructed state $(x_{2,t})$ (carbone dioxide) given by DNNO and b) pressure drop $(x_{3,t})$ derived by non parametric estimation procedure

c) Real Data Adjustment: Biological Filter problem

The popularity of the biofiltration has been increasing due to the high amount of pollutants produced by the industrial and human activity and the advantages offered by this technology. A lot of studies were conducted to enhance the performance of biofiltration systems. Consequently, there is a necessity to develop methodologies enabling to predict and determine the reactor performance, not only to design more efficient reactors but also for predicting and control the behaviour of the systems under different conditions of pollutants feeding, air flows, humidity and biomass production. The model inputs were the carbon dioxide production $\left(x_{2,t}\right)$ and the pressure drop $\left(x_{3,t}\right)$ (on line measurements), the output variable was the elimination capacities (CE), which describe the amount of pollutant eliminated by microbial activity. The DNN observer was used here to reconstruct both on line measurements, so this approach can be used like input identifier and to reconstruct the relationship between the plant states and the input variables (Fig. 3).

4. Conclusions

The novel approach suggested in this work solves one of the most important problems related with the, so-called, differential neural networks: the bounded property of the dynamic evolution for the weights parameters. The asymptotic convergence has been demonstrated applying a Lyapunov analysis, generating the corresponding conditions for the possible learning rate function. Using these conditions, two different suggestion were made in order to show why is this method feasible. Three numerical examples show the simulation efficiency for this new kind to treat the learning procedure in DNN.

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